Mathematical Software - Retake

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For exercises 1 and 2 submit a .tex and a .pdf file. For exercises 3 and 4 submit your Sage code either in text format (.txt or .sage) or as a Jupyter Notebook file (.ipynb).

Exercise 1

Write a short Latex document that contains the following theorem-like environments using the `\newtheorem` command of the `amsthm` package (the box around the text is not needed):

Proposition 1 (Fundamental Theorem of Algebra)**.** *Let $p(x)$ be a non-constant polynomial with coefficients in $\mathbb{C}$. Then there is $z \in \mathbb{C}$ such that $p(z) = 0$.*

Remark 1. Proposition 1 is not true for polynomials with coefficients in $\mathbb{R}$. For example

$$p(x) = x^2 + 1 \tag{1}$$

does not have real roots.

Theorem. *If X and Y are σ-finite measure spaces and $f : X \times Y \to \mathbb{R}$ is measurable and such that*

$$\int_{X\times Y} |f(x,y)|\mathrm{d}(x,y) < \infty$$

then

$$\int_X \left(\int_Y f(x,y)\mathrm{d}y \right) \mathrm{d}x = \int_Y \left(\int_X f(x,y)\mathrm{d}x \right) \mathrm{d}y = \int_{X\times Y} f(x,y)\mathrm{d}(x,y)\,. \tag{2}$$

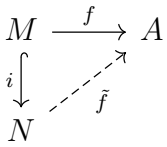
Remark 2. In practice, equation (2) means that we can switch the order of integration in a double integral.

Notice that Propositions, Remarks and some of the equations are numbered, and some of them are referred to in the Remarks. This numbering should change accordingly if more numbered Theorems and equations are added before this part of the text.

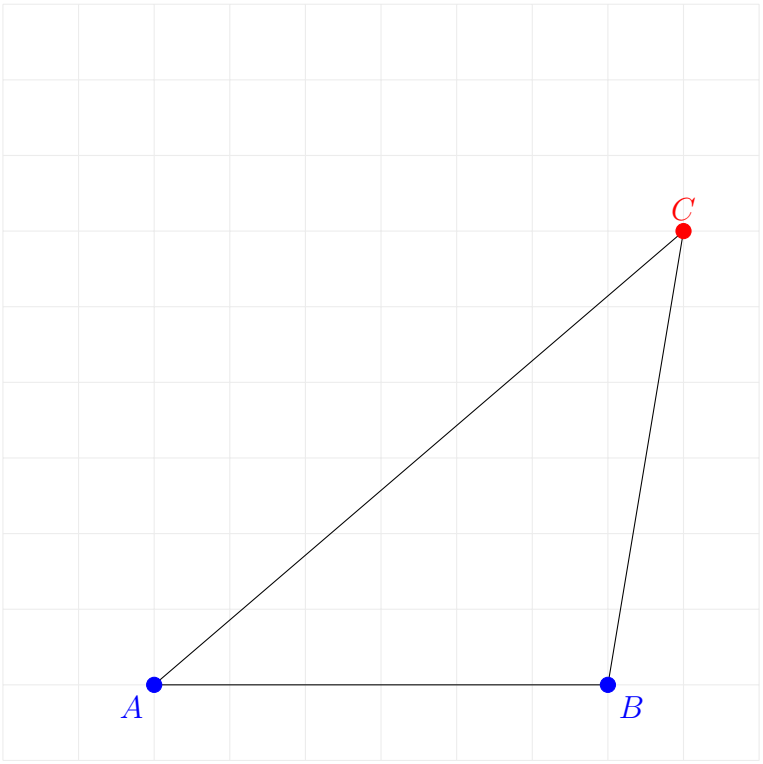
Exercise 2

Create a Latex document containing the following pictures:

(a) The following commutative diagram:



(b) A triangle with verteces on a grid, as below. Moreover, the position of the vertex C below must be easy to change at will: you should use the `\pgfmathsetmacro` command to set a value for its coordinates at the beginning, so that changing only those numbers makes the whole picture change accordingly (sides, dots, labels).



Exercise 3

Use SageMath to solve the following problems:

(1) Find the roots of the following polynomial over $\mathbb{Q}$, over $\mathbb{R}$ and over $\mathbb{C}$:

$$p(x) = x^6 + x^5 - 2x^4 - 3x^3 - x^2 + 2x + 2$$

(2) Find the determinant, the trace and the characteristic polynomial of the following matrix:

$$A = \begin{pmatrix} 2 & 3 & 0 & 1 & 2 \\ 1 & 0 & \frac{1}{2} & 1 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & 0 \end{pmatrix}$$

(3) Find the solutions of the linear system $A\mathbf{x} = \mathbf{0}$, where A is the matrix above and $\mathbf{0}$ is the zero vector.

(4) Find the points of intersection of the circle of equation $x^2 + y^2 = 4$ ad the ellipse of equation $\left(\frac{x}{2}\right)^2 + (2y)^2 = 4$

(5) Plot the graph of the function $f(x) = \sqrt{1 - x^6}$.

(6) Find the area between the x-axis and the grap of the function of the previous point.

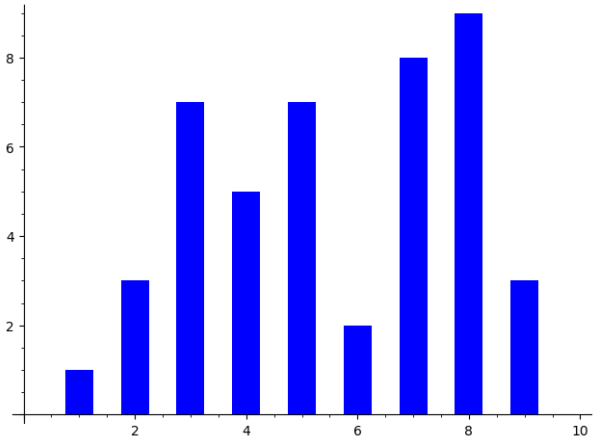
(7) Find the derivative, a primitive (integral) and the Taylor series expansion up to order 4 of the function $h(x) = \log(1 + x + x^2)$.

(8) Use Sage to get the Latex code for the objects you computed in the previous point.

(9) Find a solution for the differential equation with initial conditions

$$\begin{cases} g'(x) &= \frac{1}{3}g(x) - 7 \\ g(1) &= 30 \end{cases}$$

(10) Draw a bar chart of the data set $[0, 1, 3, 7, 5, 7, 2, 8, 9, 3]$, like the foll following:



Grading

Exercise 1 (5 points).

- A correct use of the `\newtheorem` command is worth 2 out of 5 points.
- A correct use of the labelling and reference system is worth 2 points.
- Reproducing correctly the mathematical formulas is worth 1 point.

Exercise 2 (5 points).

- Part (a) is worth 2 points: 1 point for having the letters M, N and A in the correct position and the arrows pointing between them and 1 point for the style of the arrows and the labels f, i and $\tilde{f}$ in the correct position.
- Part (b) is worth 3 points: 1 point for drawing a triangle, 1 point for the other decorative elements (colored dots, grid lines and labels) and 1 point for having the point C correctly set as a macro so that it can be changed easily.

Exercise 3 (10 points).

- Each of the 10 parts is worth 1 point.